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Performance analysis and optimization of elliptic fins circumscribing a circular tube

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Abstract

The performance of elliptic disc fins has been analyzed using a semi-analytical technique. It has been shown that the efficiency of such fins can also be predicted very closely using the sector method. However, the equivalent annulus method is not suitable for this fin geometry. A method for the optimum design of fins, using a constraint of either fin volume or rate of heat dissipation has also been suggested. Optimum elliptical fins dissipate heat at a higher rate compared to an annular fin when space restriction exists on both sides of the fin. Even when the restriction is on one side only, the performance of elliptical fin is comparable to that of eccentric annular fin for a wide parametric range.

Keywords: Elliptic disc fin; Fin performance; Optimization; Semi-analytical technique; Annular fin; Eccentric annular fin; Space restriction

1. Introduction

Fins or extended surfaces are frequently used in heat transfer equipments to increase the surface area, and, consequently, to augment the rate of heat transfer between the primary surface and the surrounding fluid. The selection of any particular type of fin depends mainly on the geometry of this primary surface. Radial or annular fins are one of the most popular choices for enhancing the heat transfer rate from the primary surface of cylindrical shape. It is well known that the rate of heat transmission from the fin decreases with the increase of fin length and hence, the entire heat transfer surface of a fin may not be equally utilized. For this reason, there is a continuous effort by the designers to determine the optimum fin that will maximize the rate of heat transfer for a specified fin volume or minimize the fin volume for a given heat duty. This fin optimization problem can be classified into two categories. In the first category, for a desired heat transfer, the fin shape (or fin profile) and all its dimensions are determined in such a way that the amount of material required is the minimum. Alternatively, given a fin profile, dimensions are obtained that satisfy the optimization conditions. The resulting fin shape obtained using the first category of the optimization technique is in general curved [1–7].

The resulting fin profile achieved through the first criterion of the above two optimization techniques is always difficult to manufacture. Moreover, the fins with optimum profile are long and narrow; they need larger space and are weak near the tip. From the application point of view, the second kind of optimization approach is more popular. Following this approach attempts have been made to design fins with triangular as well as trapezoidal profile to gain a shaving in fin material. Smith and Sucec [8] derived analytically the efficiency of circular fins of triangular profile by using Frobenius method. Kundu and Das [9] addressed the analytical solution for the performance and optimization of straight taper fins with variable heat

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Nomenclature

a, b	length of the semi-major and semi-minor axes respectively of an elliptical fin (m)
A, B	dimensionless major and minor axes lengths a/r_i and b/r_i respectively
B_0	dimensionless parameter used in Eq. (13)
Bi	Biot number based on the surface heat transfer coefficient, hr_i/k
Bi^t	Biot number based on the tip heat transfer coefficient, $h^t r_i/k$
C_j	dimensionless unknown constants, determined from Eq. (6)
e	eccentricity (m)
E_{ij}	defined in Eq. (7)
F	hyper geometric function
F_i	defined in Eq. (8)
G	variable defined in Eq. (9)
h	surface convective heat transfer coefficient (W/m^2K)
h^t	convective heat transfer coefficient at the tip (W/m^2K)
$I_m(Z)$	modified bessel function of first kind of order m and argument Z
k	thermal conductivity of the fin material (W/mK)
$K_m(Z)$	modified bessel function of second kind of order m and argument Z
n	total number of nodal points taken on the tip boundary
q	actual heat transfer rate through the fin (W)
Q	dimensionless actual heat transfer rate, $q/[4\pi k r_i(T_b - T_a)]$
q_i	ideal heat transfer rate (W)
Q_i	dimensionless ideal heat transfer rate, $q_i/[4\pi k r_i(T_b - T_a)]$
r	radial distance starting from the tube centre (m)
R	dimensionless radial distance, r/r_i
R_S	space restriction, see Fig. 1(b) and (c)
r_i	outer radius of the tube (m)
r_t	radial tip distance from the centre of the tube (m)
R_t	dimensionless radial tip distance, r_t/r_i
t	semi-thickness of an elliptic fin (m)
T_a	surrounding ambient temperature (K)
T_b	fin base temperature (K)
U	dimensionless fin volume, $v/2\pi r_i^3$
v	fin volume (m^3)
Z_0	dimensionless fin parameter, $\sqrt{Bi/\xi}$

Greek symbols

β	dimensionless tip loss, $Bi^t\xi/Bi$
ε	dimensionless eccentricity, e/r_i
η	fin efficiency
ϕ	angular position measuring from the tube centre (rad)
Ω	fin effectiveness
ψ_i	angle, defined in Eq. (10) (rad)
θ	dimensionless fin surface temperature, $(T - T_a)/(T_b - T_a)$
ξ	dimensionless thickness, t/r_i

transfer coefficient. Aziz [10] demonstrated thoroughly the steps for the determination of the optimum dimensions of fins for all types of common profile shapes.

Although fins with a variable thickness economizes the use of costly fin material, the constant thickness fins are extensively used in heat exchange applications mainly due to the ease of manufacturing and for simple design. The thermal analysis of concentric annular fins with constant thickness was first investigated by Harper and Brown [11]. Brown [12] determined the optimum dimensions of this fin. Kundu and Das [13] proposed a step change in thickness for radial disc fins to achieve material saving. On the other hand, where, a restriction of space exists on one side of the tubes or there exists an angular variation of tube temperature, fins may be designed with an eccentric annular shape [14] for a better dissipation of heat.

Circular or concentric annular geometry is mostly preferred in heat transfer equipment design. They are not only easy to fabricate but their analysis is also simple due to a radial symmetry. However, shapes with a radial symmetry are always the best choice. For example, the pressure drop due to flow past circular tubes can be reduced if tube shape is changed to elliptic one. The performance of fin-tube heat exchangers with elliptic tubes was first studied numerically by Brauer [15]. Recently, Kundu et al. [16] have determined the performance and optimum dimensions of plate fin circumscribing elliptic tubes by using FEM technique. Lin and Jang [17] analyzed numerically the efficiency of elliptic fin circumscribing an elliptic tube under dry, partially wet and fully wet conditions.

In a two-dimensional plane, when a space restriction is there on one particular side, one can select eccentric annular fins [14] in lieu of concentric annular fins and can still reduce the weight of the design. On the other hand, if space restriction is there along one particular direction, while the perpendicular direction is relatively unrestricted elliptic fins could be a good choice for material saving. Elliptic fins posses other advantages also. Thermal designers face a challenge of accommodating continuously increasing power density within a given volume and envelope shape. This demands non-conventional and innovative design of heat sinks. However, to the best of the authors' knowledge, 2D analysis of elliptic fins circumscribing circular tubes has not been attempted so far. This has motivated the present investigation.

The first objective of the present work is to present a well posed mathematical model for determining the temperature field in the elliptic fin using a semi-analytical technique. Fins with constant base temperature and with heat dissipation from the fin surface to surroundings solely by convection have been considered in this analysis. The present semi-analytical model has been validated with the existing closed form solution for the limiting case of concentric annular disc fin [12]. The fin performance of the elliptic fin also predicted by the equivalent annulus and sector method has been compared with the present method of solution. It has been observed that the result of the sector method gives slight under prediction whereas the result calculated from the equivalent annulus method show over prediction. A comparative study has also been carried out between the fin performance of elliptic fins and with the eccentric [14] and concentric [12] disc fins.

The second objective is to develop a comprehensive scheme for optimization of elliptic fin. The optimization has been presented in a general form in which either the rate of heat transfer or the fin volume is considered as a constraint while the eccentricity is specified a priory. Optimization of elliptic fin has been demonstrated for a restricted fin dimension on both side of the tube also. Finally, a comparative study of the heat transfer through an elliptic fin and with both a concentric and an eccentric disc fins for an optimal design condition has been made. The results show that the elliptic fin is a better choice for transferring heat with an identical fin volume and eccentricity. For a constant space restriction on one side of the tube, the heat transfer from the optimum elliptic fin is marginally lower in comparison with the optimum eccentric fin. However, for a space restriction on both sides, the elliptic fin is the always better option for transferring heat with respect to concentric fin.

2. Mathematical model

To analyze the heat transfer from an elliptic fin circumscribing a circular tube (Fig. 1), certain idealizations are made. It is assumed that the fin exchanges heat with ambient medium (existing at a constant temperature) solely by convection and the convective coefficient is uniform along the fin surface. Further, there is no temperature gradient normal to the fin surface. Based on the above idealizations, the steady state energy equation for constant conductivity of the fin material and uniform temperature at the fin base can be written in dimensionless form in a cylindrical polar coordinate system as:

$$\left[\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)+\frac{1}{R}\frac{\partial^2\theta}{\partial\phi^2}\right]=Z_0^2R\theta \tag{1}$$

From Fig. 1, it can be seen that the axes of the elliptic fin divides it into of four geometrical symmetric modules. Based on the assumptions made above, the geometrically symmetric modules are also symmetric from heat transfer point of view. It is sufficient to analyze any one of these modules [as shown in Fig. 1(d)] for determining the

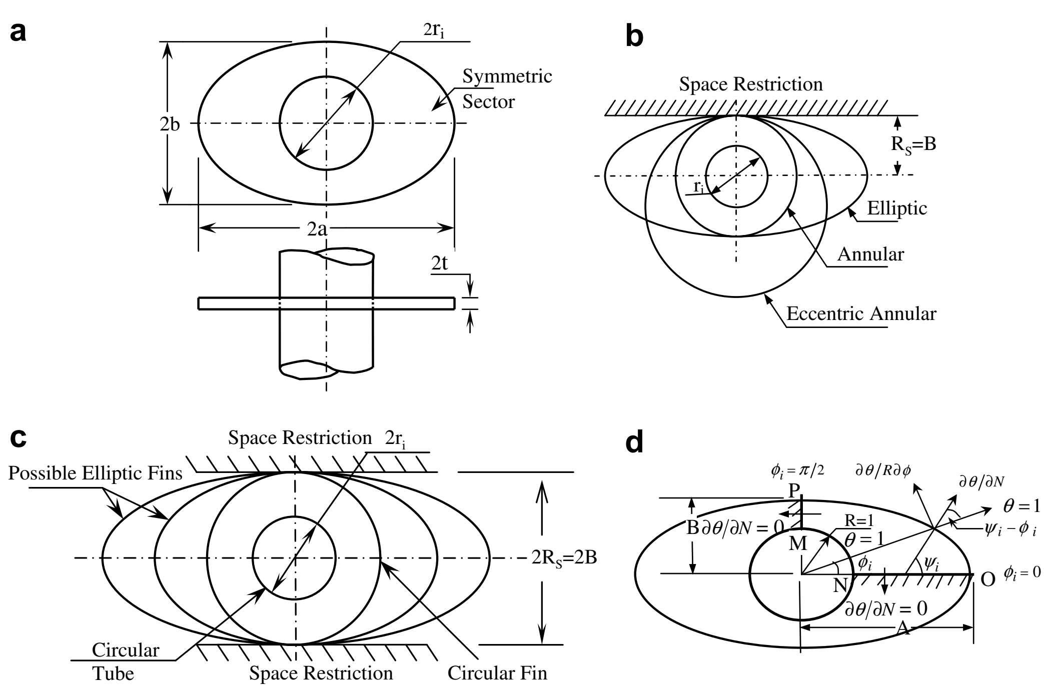

Fig. 1. Circumferential fin circumscribing a circular tube: (a) Schematic diagram of an elliptic fin; (b) Probable fin geometries in case of restriction on one side; (c) Possible concentric and elliptic fins for a space restriction on both sides of the tube and (d) Computational domain MNOPM with the boundary conditions.

performance of the fin as a whole. The physical boundary conditions are taken as follows: the temperature of the outer periphery of the tube MN is constant; there is no net heat conduction across the section NO and MP due to symmetry. Either insulated or convection heat transfer may be considered at the fin tip OP. Thus, these boundary conditions can be mathematically expressed as follows:

$$\theta = 1 \quad \text{at } R = 1 (0 \leqslant \phi \leqslant \pi/2) \tag{2a}$$

$$\partial\theta/\partial\phi = 0 \quad \text{at } \phi = 0 (1 \leqslant R_t \leqslant A) \tag{2b}$$

$$\partial\theta/\partial\phi = 0 \quad \text{at } \phi = \pi/2 (1 \leqslant R_t \leqslant B) \tag{2c}$$

and

$$\partial\theta/\partial N = -\beta Z_0^2 \theta \quad \text{at } R = R_t (0 \leqslant \phi \leqslant \pi/2) \tag{2d}$$

where

$$R_t = AB \Big/ \sqrt{A^2 \sin^2\phi + B^2 \cos^2\phi} \tag{3}$$

and

$$\beta = Bi^t \xi / Bi \tag{4}$$

It may be noted that, Eqs. (2a)–(2c) can be satisfied exactly at the physical boundaries in the polar coordinate system. However, neither polar nor cartesian coordinate can exactly satisfy the boundary condition at the fin tip. Nevertheless, the fourth boundary condition (2d) can be satisfied at a large number of discrete points along this boundary. With the first three boundary conditions Eqs. (2a)–(2c), Eq. (1) can be solved by employing the separation of variables as

$$\theta - \frac{I_0(Z_0 R)}{I_0(Z_0)} = \sum_{j=1}^{\infty} C_j \left[\frac{\cos\{2\phi(j-1)\}}{I_{2j-2}(Z_0)}\right] \begin{vmatrix} I_{2j-2}(Z_0) & I_{2j-2}(Z_0 R) \\ K_{2j-2}(Z_0) & K_{2j-2}(Z_0 R) \end{vmatrix} \tag{5}$$

The above temperature field is dependent on the constants "C_j". For the determination of these constants, Eq. (2d) can be employed at discrete points along the fin tip and it can be expressed as

$$\sum_{j=1}^{\infty} E_{ij} C_j = F_i \quad \text{for } i = 1, 2, 3, \ldots, \infty \tag{6}$$

where

$$E_{ij} = \begin{vmatrix} 0 & -Z_0 R_t \cos(\psi_i - \phi_i) & G \\ I_{2j-2}(Z_0) & I_{2j-2}(Z_0 R_t) & I_{2j-1}(Z_0 R_t) \\ -K_{2j-2}(Z_0) & -K_{2j-2}(Z_0 R_t) & K_{2j-1}(Z_0 R_t) \end{vmatrix} \tag{7}$$

$$F_i = \frac{Z_0}{I_0(Z_0)} \begin{vmatrix} \cos(\psi_i - \phi_i) & -\beta Z_0 \\ I_0(Z_0 R_t) & I_1(Z_0 R_t) \end{vmatrix} \tag{8}$$

$$G = 2(j-1)\{\cos(\psi_i - \phi_i) - \tan[2\phi(j-1)]\sin(\psi_i - \phi_i)\} + Z_0^2 R_t \beta \tag{9}$$

$$\psi_i = \tan^{-1}(A^2 \tan\phi_i / B^2) \tag{10}$$

and the suffix 'i' is the ith point on the boundary. Theoretically, infinite numbers of points are required to satisfy Eq. (6) for obtaining the closed form result. However, in practice, a finite number of equations are only considered. The number (say n) is chosen in such a way that the final result yield a desired accuracy. To determine the unknown constants, the algebraic equations can be solved by Gauss–Ellimination method. After getting the unknown constants, one can calculate the dimensionless heat transfer rate Q as

$$Q = \frac{q}{4\pi k r_i (T_b - T_a)} = \xi[C_1 - Z_0 I_1(Z_0)]/I_0(Z_0) \tag{11}$$

Ideal or maximum possible rate of heat transfer from the fin Q_i is calculated if the fin surfaces were maintained at its base temperature. In dimensionless form this can be expressed as

$$Q_i = \frac{q_i}{4\pi k r_i (T_b - T_a)} = \frac{Bi(AB-1)}{2} + \frac{2BiA\beta}{\pi} \int_{\phi=0}^{\pi/2} \frac{d\phi}{\sqrt{1 - B_0 \sin^2\phi}} \tag{12}$$

where

$$B_0 = \varepsilon^2/(\varepsilon^2 - 1) \tag{13}$$

and

$$\varepsilon = \sqrt{1 - B^2/A^2} \tag{14}$$

Eq. (12) can be integrated numerically using the Simson's 1/3 rule to yield the ideal heat transfer rate. However, an analytical expression for the ideal heat transfer rate can be obtained by integrating the above elliptic integral equation as

$$Q_i = Bi\left[\frac{(AB-1)}{2} + A\beta F\left(\frac{1}{2}, \frac{1}{2}; 1; B_0\right)\right] \quad \text{for } |B_0| < 1 \tag{15}$$

where F is the hyper geometrical function [18].

Fin efficiency is expressed conventionally as

$$[\eta] = [Q/Q_i] \tag{16}$$

Fin effectiveness can be expressed as follows:

$$\Omega = [C_1 - Z_0 I_1(Z_0)][\xi Z_0^2 I_0(Z_0)] \tag{17}$$

3. Optimization

The volume of an elliptic fin subscribing a circular tube is given by

$$U = \frac{V}{2\pi r_i^3} = (AB-1)\xi \tag{18}$$

From Eqs. (11) and (18), it is perceived that both the dimensionless heat transfer (Q) and fin volume parameter (U) are dependent on the design variables ξ, A and B. It is of interest to note that the elliptic fin is converted to a circular fin at the optimum design condition if both the design parameters A and B are varied. Therefore, the optimization of elliptic fins is possible only if either A or B is specified a priory. It has been discussed earlier that, if there is a space restriction imposed on both sides of the tube, the freedom of varying the minor axis of the elliptic fin gets restricted (see Fig. 1(b) and (c)). Accordingly the dimension B is considered as the design constant. Using Lagrange multiplier technique, the optimality criteria [19] is obtained for given thermo-geometric parameters and design constraint B as follows:

$$[\partial Q/\partial A][\partial U/\partial \xi] - [\partial Q/\partial \xi][\partial U/\partial A] = 0 \tag{19}$$

Using Eqs. (11) and (18), the above equation can be written as

$$2\xi \frac{\partial C_1}{\partial \xi} - \frac{2(AB-1)}{B}\frac{\partial C_1}{\partial A} + \frac{Q[2I_0(Z_0) + Z_0 I_1(Z_0)]}{\xi} + 2Z_0 I_1(Z_0) + Z_0^2 I_2(Z_0) = 0 \tag{20}$$

The derivative of C_1 may be determined by differentiating Eq. (6)

$$\begin{cases} [\partial E_{ij}/\partial \xi][C_j] + [E_{ij}][\partial C_j/\partial \xi] = [\partial F_i/\partial \xi] = 0 \\ [\partial E_{ij}/\partial A][C_j] + [E_{ij}][\partial C_j/\partial A] = [\partial F_i/\partial A] = 0 \end{cases} \tag{21}$$

For the determination of roots of Eq. (20), one constraint equation is necessary. Either heat transfer (Eq. (11)) or fin volume (Eq. (18)) can be taken as a constraint. The generalized Newton-Raphson method [20] is used to determine the optimum dimensions. To perform this calculation, it is required to determine the values of $\partial^2 C_1/\partial \xi^2$, $\partial^2 C_1/\partial \xi \partial A$ and $\partial^2 C_1/\partial A^2$ in the every iteration. These values are obtained by differentiating Eq. (21) with respect to ξ and A. And the systems of simultaneous linear equations are solved. In each iteration, the initial guess values of the roots are chosen in such a way that the condition of convergence has been satisfied [20]. The above process can be repeated till the root is obtained to a desired accuracy. In this study, the value of optimum dimensions has been calculated considering significant values upto six decimal places.

4. Results and discussion

Based on the above analysis, the performance of elliptical fins has been estimated for a wide range of thermo-geometric parameters. Some of the salient results are presented in this section.

At the outset, an attempt has been made to examine the validity of the semi-analytical technique adopted in the present work. Unfortunately, there is no analytical result for the performance of 2D elliptical fin. However, in the limiting case, an elliptical fin becomes a concentric annular fin when the semi-minor axis approaches the semi-major axis. For this case, closed form analytical expression is available. A comparison between the closed form expression and the results obtained from the semi-analytical technique depicts an excellent agreement (Fig. 2).

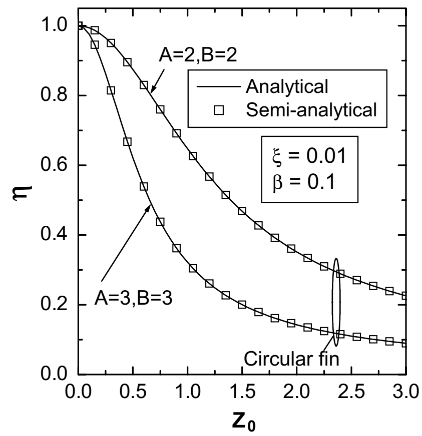

Fig. 2. Comparison of efficiency of circular fins obtained from the analytical and semi-analytical methods.

The solution of Eq. (5) enables one to find out the temperature at any point of the elliptic fin. The isotherms in the elliptical fins for different geometry are shown in Fig. 3. Though the radial symmetry of the temperature field disappears slightly away from the tube wall, the field is symmetric about the axes. The occurrence of extrema for different isotherms on the major axis depicts a typical characteristic of this unique fin geometry. It is interesting to note that, though the isotherms have their extremum on the major axis, their trend changes as one move from the fin base to fin tip along this axis. On the other hand, slightly away from the base the isotherms never show a tendency to close on the minor axis. This clearly brings out the two-dimensional effect in heat conduction.

Next, the results of the semi-analytical technique have been compared with those obtained from the approximate techniques. For the estimation of fin performance, two approximate techniques namely equivalent annulus method and sector method [21] are commonly used. Both of them have been tried in the present case and the results are shown in Fig. 4. As the fin parameter Z_0 increases, the equivalent annular technique grossly over predicts the results obtained from the semi-analytical technique. On the contrary, the result obtained from the sector method

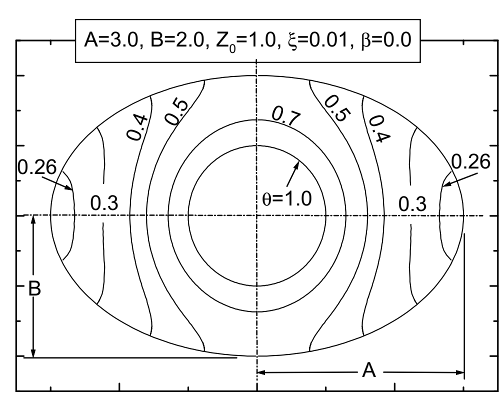

Fig. 3. Isotherms in an elliptic fin for $A = 3.0$ and $B = 2.0$.

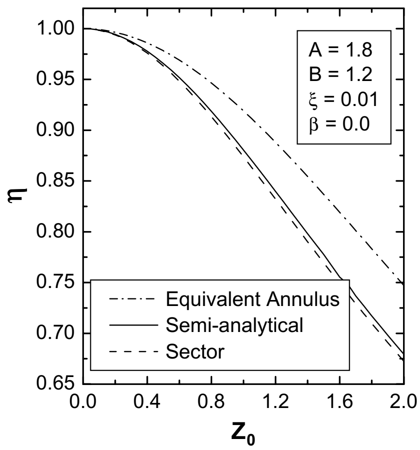

Fig. 4. Efficiency of an elliptic fin predicted by different methods.

closely follows the semi-analytical technique. As the sector method slightly under predicts the results it can safely be used for design calculations. It may be noted that the calculation by the sector method has been done by the technique modified by Kundu and Das [22].

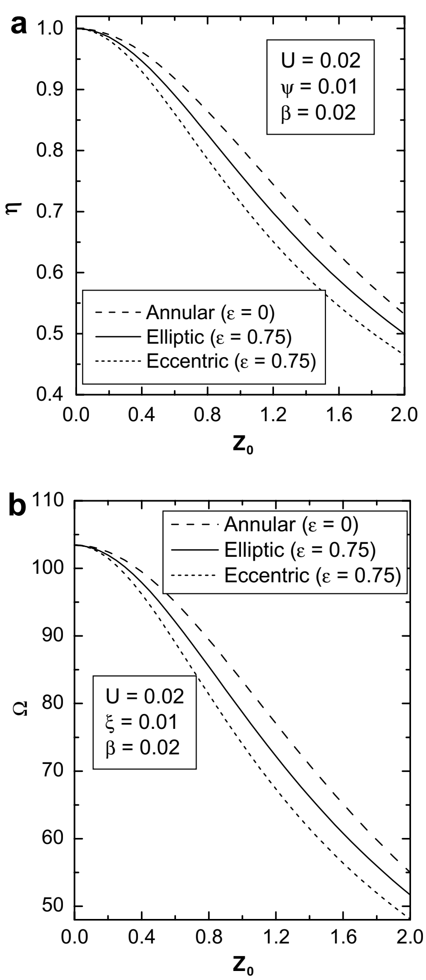

Fig. 5. Comparison of the fin performance of different fins for the same surface area and thermo-physical parameters: (a) Fin efficiency and (b) Fin effectiveness.

A comparison of efficiencies of three different fins: annular, elliptical and eccentric annular has been presented in Fig. 5(a). The eccentricity of both the elliptic and eccentric fins has been taken as 0.75. It may be noted that the geometric definitions of the eccentricity for ellipse and eccentric annulus are different. For an ellipse, it is a function of minor and major axes while for an eccentric annulus; it is given by the centre distance of the inner and outer circles [14]. For identical operating conditions, the efficiency of the annular fin is the maximum while it is a minimum for eccentric annular fin. Similar conclusion can be drawn when the effectiveness of the three fins is compared (Fig. 5(b)). This clearly depicts that for plate fin circumscribing circular tubes the loss of symmetry has an adverse effect on fin efficiency. As the eccentric annular fin has only one axis of symmetry it has the least efficiency. It may not be out of place to mention that in fin-tube heat exchangers the highly symmetric equilateral triangular array of tubes results in a higher efficiency compared to other arrangements. The geometric and thermal symmetries are therefore

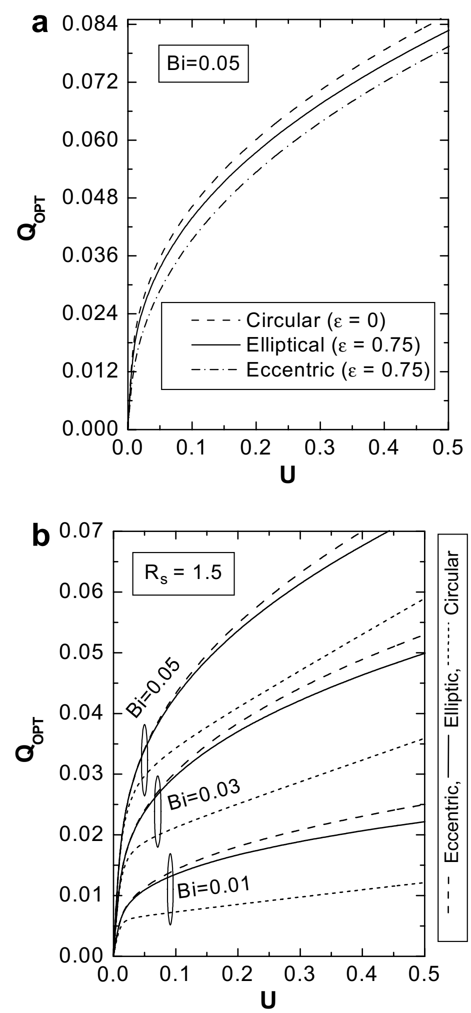

Fig. 6. (a) Comparison of heat transfer from different optimum fins as a function of U; (b) Comparison of heat transfer from different optimum fins for a given space restriction.

to be exploited fully unless there are constraints due to process requirement and manufacturing considerations. Different parametric values do not change the nature of the curves shown in Fig. 5, though the quantitative values of the performance parameter change.

In the rest of this section the characteristics of the optimum fins are presented. Fig. 6(a) depicts the variation of the optimum rates of heat transfer from the three different fins for identical fin volumes. The eccentricity of the non-circular fins has been taken as 0.75. In this case also, the annular and eccentric fins depict the maximum and minimum heat transfer rates respectively. The analysis is also made imposing a space restriction on one side of the fin ($R_S = 1.5$) and the results are shown in Fig. 6(b). It may be noted that the rate of heat dissipation is comparable for the two non-circular fins and is substantially higher than that of the annular fin. Moreover, this difference increases with the increase of the fin volume. Similar trend is obtained for other parametric ranges. This result clearly justifies the design of non-circular fins in the presence of space restriction.

The variation of the performance of two types of non-circular fins with eccentricity is given in Fig. 7. It can be seen that the performance of elliptic fins remains almost constant for a large range of eccentricity and then falls drastically. Moreover, the performance of the elliptic fin is superior to that of the eccentric fin up to a value of eccentricity approximately 0.9. It may be noted that the geometrical definition of eccentricity for these two types of fins are different. A value of eccentricity of 0.9 reduces the minor axis of the elliptic fin to such an extent that along this axis the fin tip comes very close to the fin base. An eccentric fin also suffers from a loss of symmetry and deterioration in heat transfer with the increase of eccentricity. However, the thermal degradation of the eccentric fin is not so drastic at this range of eccentricity for the other parametric values chosen in the example. This example clearly brings out the large design range available for elliptic fins and the limiting condition where the use of such fin may not be justifiable.

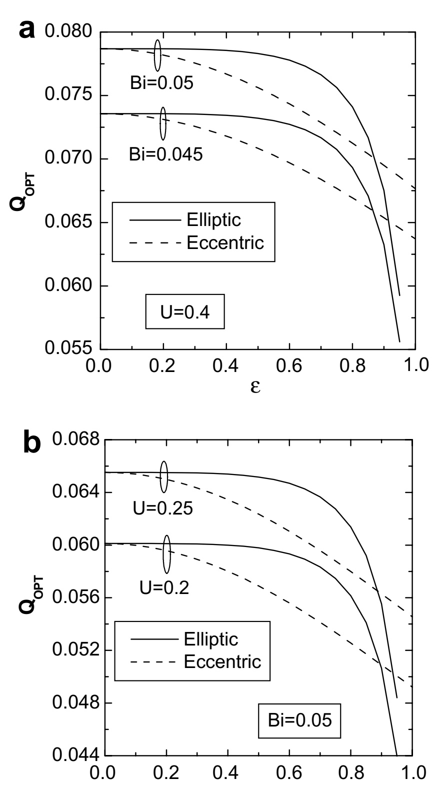

Fig. 7. Optimum elliptic and eccentric fins as a function of eccentricity for: (a) $U = 0.4$; (b) $Bi = 0.05$.

5. Conclusions

The heat transfer from elliptical disc fin circumscribing circular tubes has been analyzed in the present work. A semi-analytical technique has been adopted to determine the temperature field in this unique geometry. It has also been demonstrated that the sector method, which is an approximate technique gives a close but conservative estimate of the fin efficiencies and can safely be used for design calculations.

A calculus-based technique has been used for the optimum design of elliptical fins. By this approach one can design either a minimum volume fin for a given rate of heat dissipation or can maximize the heat transfer rate for a given fin volume. It has been shown that the elliptical fin competes well with the eccentric annular fin for certain range of thermo-geometric parameters, where there exists a space restriction on one side of the fin. Rate of heat dissipation from an optimum elliptical fin can be substantially higher than that from concentric annular fin when space restriction exists on both sides of the fin. It has also been demonstrated that the advantage of selecting an elliptic fin in lieu of a concentric circular fin becomes a maximum at a particular fin volume.

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